UNIT 3: Quadratic Equations and Quadratic Functions

Specific Objectives:

1. To acquire skills in solving quadratic equations by the method of completing the square and by formula.
2. To determine the nature of roots of quadratic equations.
3. To find the maximum and minimum values of quadratic functions.

Detailed Content	Time Ratio	Notes on Teaching
3.1 Solution of Quadratic Equations	8 *	The technique of the method of completing the square should be introduced beginning with examples like $x^2-8x+9=0$ and then progressing to examples like $3x^2-6x-14=0$ in which the coefficient of x^2 is not unity. When students have mastered the technique, the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ for solving $ax^2+bx+c=0$ ($a\neq 0$) can be derived by the same method. Students should have no difficulty in applying the quadratic formula. They should then investigate the relationships between the sum and product of the roots, α, β and the coefficients a, b, c of the quadratic equation. The relations $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$ should be memorized and then applied to exercises on calculating values of expressions such as $\frac{1}{\alpha}+\frac{1}{\beta}$, $\alpha^2+\beta^2$ and $\alpha^3+\beta^3$, and exercises on the formation of quadratic equations. Teachers should discuss the solution of simultaneous equations involving one linear and one quadratic. Both algebraic method and graphical method should be introduced. The graphical method is particularly useful in explaining why some quadratic equations have two distinct roots, two identical roots or no real roots at all. The geometrical interpretation provided by the graphical method should be emphasized.
3.2 Nature of roots	4	Teachers should guide students to discover that the nature of roots of the quadratic equation $ax^2+bx+c=0$ is determined by the discriminant $D=b^2-4ac$. Students should be able to distinguish whether the roots are real or complex (unreal); equal or unequal; rational or irrational. For example, the discriminat of the

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		equation $(b-c)x^2+(c-a)x+(a-b)=0$, where a, b and c are distinct integers, is $(c-a)^2-4(b-c)(a-b)=(a+c-2b)^2$. This implies the roots are rational. Students are also expected to see that the roots are equal if $a+c-2b=0$. Relevant exercises should be given. The following is an example. *Example* Find the range of values of m for which the roots of the following equations are real. $y=m(x+2)$ $y^2=8x$ In this example, students are expected to obtain the equation $m^2x^2+(4m^2-8)x+4m^2=0$ and the condition $(4m^2-8)^2-16m^4\geq 0$.
3.3 Quadratic Functions	4	Students should be able to express ax^2+bx+c in the form $a(x+\frac{b}{2a})^2+\frac{4ac-b^2}{4a}$ by the method of completing the square. They should be guided to determine that the maximum value (if a<o) or the minimum value (if a>o) of ax^2+bx+c is $\frac{4ac-b^2}{4a}$, which occurs at $x=-\frac{b}{2a}$. Students should also know that the line of symmetry is $x=-\frac{b}{2a}$. Teachers may discuss the graph of the quadratic function ax^2+bx+c with students and lead them to summarize the following cases for various values of a, b and c. <table><tr><th>a</th><th>b</th><th>c</th></tr><tr><td rowspan="2">>0</td><td>>0</td><td>>0</td></tr><tr><td>=0</td><td>=0</td></tr><tr><td rowspan="2"><0</td><td><0</td><td><0</td></tr></table>

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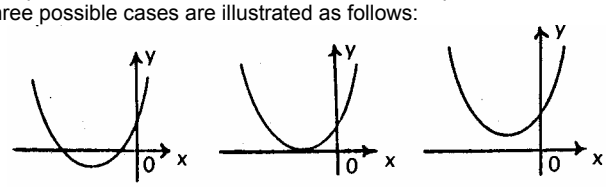
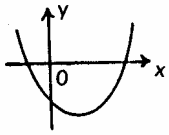
		In each of the 18 cases, there may be one or three cases corresponding to one or three possible values of the discriminant. For example, when a>0, b>0, c>0, the three possible cases are illustrated as follows:  $b^2-4ac>0$ $b^2-4ac=0$ $b^2-4ac<0$ but when a>0, b<0 c<0, there is only one possible case, the graph of the quadratic function is sketched below.  $b^2-4ac>0$ Students should be encouraged to make simple sketches of quadratic functions when they are solving problems involving quadratic equations or quadratic inequalities. Exercises regarding quadratic functions may include the following. *Example 1* Find the maximum or minimum of the following expressions. (*a*) $x^2 - 8x + 9$ (*b*) $6 + 6x - x^2$ (*c*) $\frac{1}{x^2 - 6x + 11}$

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		Example 2 For what values of m and n are the following expressions always positive? (*a*) $3x^2 + 2x + m$ (*b*) $nx^2 - 5x + 4$
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